

Reza N. Jazar

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# Vehicle Dynamics

Theory and Application

*Third Edition*

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Theory and Application

Third Edition

 Springer

Reza N. Jazar  
School of Engineering  
RMIT University  
Melbourne, VIC, Australia

School of Civil Engineering  
Xiamen University of Technology  
Jimei, Xiamen, China

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